Exam Seat No:_____ **Enrollment No:**_____

C.U.SHAH UNIVERSITY

Wadhwan City

Subject Code : 4SC02MTC1 **Summer Examination-2014** **Date: 26/05/2014**

Subject Name: Mathematics - II

Branch/Semester:- B.Sc(Science)/II **Time:02:00 To 5:00**

Examination: Regular

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places
(5) Assume suitable & Perfect data if needed

SECTION-I

Q-1 a) Find polar form of (i) $1+i$ and (ii) $\sqrt{3}-i$. (02)

b) Prove that $\cos h^2x - sinh^2x = 1$. (02)

c) Simplify: $\frac{(\cos 3\theta + i \sin 3\theta)^2 (\cos 4\theta - i \sin 4\theta)^{-2}}{(\cos 2\theta - i \sin 2\theta)^{-7}}$. (01)

d) Evaluate: $\int_0^{\pi/2} cos^6 x dx$. (01)

e) Define limit point of a sequence. (01)

Q-2 a) Find all fourth roots of unity and sketch them on unit circle. (05)

b) Prove that $\cos 5\theta = 16\, cos^5\theta - 20 cos^3\theta + 5 \cos\theta$. (05)

c) Prove that $(1+i)^n + (1-i)^n = 2^{\frac{n}{2}+1} cos\frac{n\pi}{4}$. (04)

OR

Q-2 a) State and prove De Moivre's theorem. (05)

b) Prove that $\cos h^{-1}(z) = \log(z + \sqrt{z^2-1})$. (05)

c) If $\log(x+iy) = 2 - \frac{3\pi}{4}i$ then find value of x and y. (04)

Q-3 a) Prove that $\int cos^n x dx = \frac{cos^{n-1}x \sin x}{n} + \frac{n-1}{n} I_{n-2}$, for $\in N$. (05)

b) Evaluate: $\int_0^{1/2} x^3 (1-4x^2)^{1/2}\, dx$ (05)

c) Using definition of limit prove that $\lim_{n\to\infty} x_n = \frac{2}{3}$, if $x_n = \frac{2n+1}{3n+5}$. (04)

OR

Q-3 a) Evaluate: $\int_0^{\infty} \frac{x^2}{(1+x^6)^{7/2}} dx$. (05)

b) Prove that $\int_0^1 x^m (1-x)^n dx = \frac{m!n!}{(m+n+1)!}$. (05)

c) Using definition of limit prove that $\lim_{n\to\infty} \sqrt[n]{n} = 1$ (04)

SECTION-II

Q-4 a) Solve: $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$ (02)

b) Write condition that the plane $lx + my + nz = 0$ be a tangent to the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$. (02)

c) Write general equation of ellipsoid. (01)

d) Which is the vertex of cone having second degree homogeneous equation in x, y and z. (01)

e) Find $\frac{1}{D^2}x^4$. (01)

Q-5 a) Solve: $(D^2 - 7D + 6)y = 2e^{3x}$, given that $y = 1, \frac{dy}{dx} = 0$ when $x = 0$. (05)

b) Solve: $y''' - 3y'' + 9y' - 27y = \cos 3x$ (05)

c) Solve: $(D^2 - 4D + 1)y = e^{2x}\sin x$. (04)

OR

Q-5 a) Solve: $(D^2 - 5D + 6)y = x + e^{4x}$. (05)

b) Solve: $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$. (05)

c) Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = 2x^2$. (04)

Q-6 a) Find equation of cone whose vertex is the point $(1, 1, 0)$ and whose guiding curve is $x^2 + z^2 = 4, y = 0$. (05)

b) Find equation of lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. (05)

c) Prove that equation of right circular cylinder whose axis is the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r is (04)

$$(x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2 - \frac{[l(x-\alpha) + m(y-\beta) + n(z-\gamma)]^2}{l^2 + m^2 + n^2} = r^2$$

OR

Q-6 a) Check whether the equation $2x^2 + 2y^2 + 7z^2 - 10yz - 10xz + 2x + 2y + 26z - 17 = 0$ represent a cone with vertex at (2,2,1) or not. (05)

b) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + 2y^2 = 1, z = 0$. (05)

c) Prove that locus of lines through the vertex of cone $ax^2 + by^2 + cz^2 + 2fyz + 2gxz + 2hxy = 0$ and perpendicular to it's tangent plane is $Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gxz + 2Hxy = 0$. (04)

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